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ADDITIONS TO THE SOLUTION OF THE STATICALLY INDETERMINATE PROBLEM OF TENSION IN BANDSAW BLADE

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ABSTRACT

At the Faculty of Forest Industry, seemingly for the first time in the bandsaw studies history, the statically indeterminate problem of spring mechanism tension in bandsaw blade sections was set up and solved. The solution, transferred and generalized for a horizontal bandsaw, was defended in a Ph.D. thesis. Questions and discussions arose necessitated additions and details to the solution spared in previous papers for their limited volume. Now, the following points are detailed: the concentration of distributed loads, the subcritical friction, and the application of L'Hôpital's rule for obtaining the tension of the blade in the state of idle running (without cutting).

Key words: bandsaw, bandsaw blade, tensioning spring mechanism, statically indeterminate problem.

INTRODUCTION

In (Stefanov 2013), the statically indeterminate problem of the tensile force in bandsaw blade sections was set and solved in the case of tensioning the blade by a spring mechanism. The equation for the statically indeterminate tensile force X in the non-cutting part of the blade of a vertical bandsaw (Fig. 1) was worked out. It is obvious that the obtained equation appeared for the first time in the bandsaw studies history thanks to an essential point: the deformation compatibility condition for solving the statically indeterminate problem was formulated. That equation for X was generalized in (Stefanov and Atanasov 2014a) while transferring it to a horizontal bandsaw (Fig. 2). It was proved that the forces P_1 and P_2 (Fig. 2) can be ignored as the error in the determination of X is significantly lower than 5 per cent. Therefore, the forces N_B and Φ, insignificantly inclined to the axis AB (Fig. 2), coincide with it, practically. Eventually, as to the problem of X determination, the scheme for the vertical bandsaw (Fig. 1) proves practically valid also for the horizontal bandsaw (Fig. 2) after an imaginary 90° turn.

The obtained equation for X (Stefanov and Atanasov 2014b) is:

$$X = \frac{1}{2d + \pi R + \dfrac{4Ebs}{c}}\left[-P\left(d_E + \frac{h_p}{2}\right) - \frac{PR}{\mu_\rho} - \frac{\pi R \Phi}{2} + 2Ebs\lambda_s - Ebs\Delta l_\theta - Ebsl\alpha_t\Delta t + \frac{2Ebs\Phi}{c}\right] \quad \textbf{(1)}$$

where:

$$\mu_\rho = \mathrm{tg}\alpha = \frac{1}{\pi}\ln\left(1 + \frac{P}{X - \Phi/2}\right). \quad \textbf{(2)}$$

The nomenclature of the symbols in these equations follows (see each of Fig. 1 and Fig. 2).

E – Young’s modulus of the blade material;

λ_s – displacement of the spring’s end held by the screw; thanks to this displacement, produced by the screw, the blade’s pretension is enabled;

c – spring’s stiffness (spring’s constant);

R, d, d_E, h_p, α – dimensions and angle shown in Fig. 1 and Fig. 2;

l – length of the blade ($l = 2d + 2\pi R$);

Δl_θ – lengthening of l along the central blade's fibre as a result from a driven wheel tilt, or both from this tilt and a driving wheel tilt if any; if the tilt causes shortening of l, then Δl_θ is taken with negative algebraic value;

Δt – heating the blade (in ^{o}C);

α_t – coefficient of free thermal expansion per 1 ^{o}C;

μ_ρ – working coefficient of friction (coherence) with the driving wheel (μ_ρ = tgα = T/N_A);

P – cutting force, longitudinal to the blade;

Φ – force of mass inertia of each of the two blade half-circumferential parts ($\Phi = 2\rho bsv^2$ where $v = \omega R$ is the blade speed and ρ is the mass density).

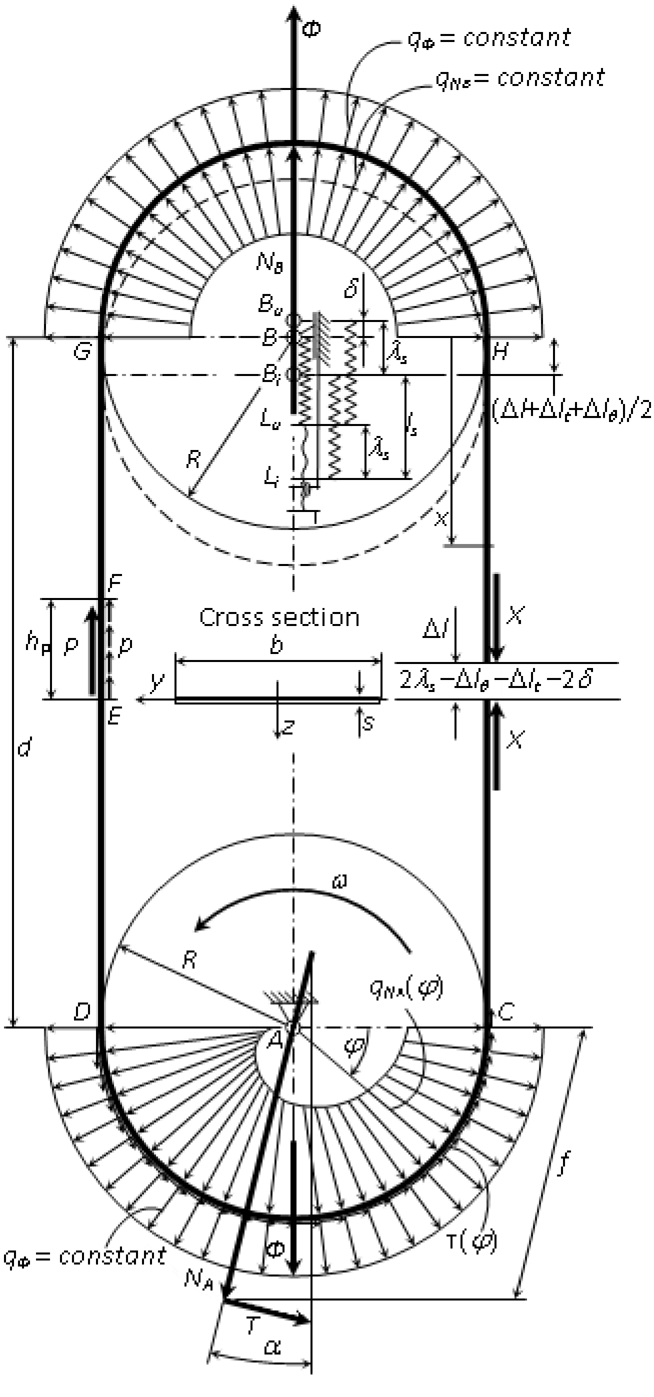

Figure 1: Scheme for determination of *X* of a vertical bandsaw

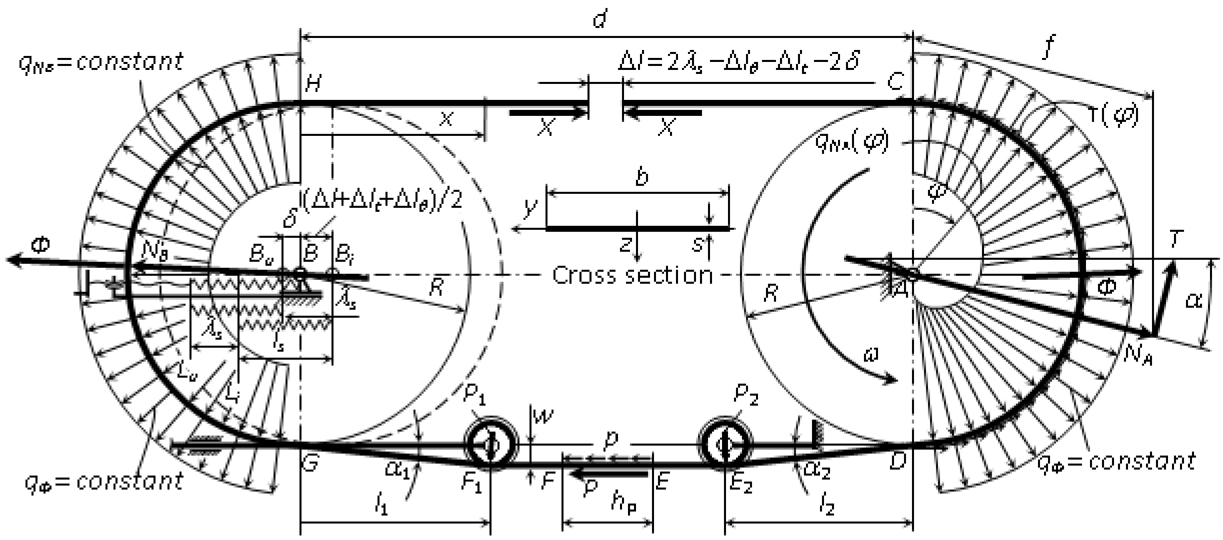

Figure 2: Scheme for determination of *X* of a horizontal bandsaw

It is to note that the working coefficient of friction was simply labelled by μ in our previous papers. Now, the subscript $_\rho$ is added to the symbol μ. The necessity of this change will be discussed below. Besides, Δl_θ is added into Fig. 1 taken from (Stefanov 2013). It is also to note, again in addition to (Stefanov 2013), that, for a vertical bandsaw, λ_s is the displacement after the spring takes the weight of the upper wheel first. This note is not very essential and can be omitted: that weight is taken by the spring with an insignificant addition to λ_s. As to the horizontal bandsaw, the spring does not take any wheel weight first.

The papers (Stefanov and Atanasov 2014a and 2014b) took an essential place in the thesis (Atanasov 2014). The main remarks made during the defence of the thesis were, as follows.

It was stated that, both for the vertical bandsaw (Stefanov 2013) and the horizontal bandsaw (Atanasov 2014), the application of the equations $N_B = q_{N_B}.2R$ and $\Phi = q_\Phi.2R$ instead of e.g. $N_B = q_{N_B}.\pi R$ and $\Phi = q_\Phi.\pi R$ was not well grounded. It was also stated that the force N_B was determined by q_{N_B} and the diameter $2R$, whereas it acts onto the half of the wheel circumference. Besides, it was questioned as unclear why the traction force T is shown at a distance f from the driving wheel center i.e. out of the places of contact between the cutting tool and the wheel periphery. As well, it was questioned how the equation for X_0 had been worked out, i.e. for X during the idle running (P = 0 and μ = 0), with regard to resolution of the indeterminate quotient [0/0] by means of L'Hôpital's rule application. With that, there was also such an objection: if assuming the working coefficient of coherence to be μ = 0, then driving the blade will not be enabled.

These remarks make clear that, regardless of the successful defense of (Atanasov 2014), it is necessary and useful to publish additions and details to the solution for X in the present paper. They were spared in our former papers for their limited volume. Now, the following points are detailed: the concentration of distributed loads, the subcritical friction, and the application of L'Hôpital's rule. This paper will also be in favor of clarification of some basic engineering mechanics knowledge applied to the bandsaw blade mechanics, and will be a new perusal of that mechanics.

1. CONCENTRATION OF A FORCE DISTRIBUTED UNIFORMLY IN RADIAL DIRECTIONS ALONG A SEMI-CIRCUMFERENCE

This problem is a particular case of the general problem of concentration of a force distributed along an arbitrary curve in a plane and having an arbitrary distribution diagram (Fig. 3a). In this general case, the distribution intensity is a vector $\vec{q}(s) = \mathrm{d}\vec{Q}(s)/\mathrm{d}s$ with variable magnitude, direction and directional sense (the intensity of the friction force distributed along the semi-circumference of the driving wheel in Fig. 1 and Fig. 2 is such a variable vector). Then, the distribution problem is not simple: all the infinitesimal forces $\mathrm{d}\vec{Q}(s) = \vec{q}(s).\mathrm{d}s$ on the linear infinitesimal elements d*s* should be formed and summed to a resultant force and a resultant moment at an arbitrary center *O*, and next, a new center *C* should be found at which the resultant concentrated force $\vec{Q}$ will only remain.

Details can be found in Chapter 4 in (Stefanov 2014). There are sums there which should be replaced by integrals now. It is to notice that $\vec{Q}$ is not obtained at all as just a product of any intensity $\vec{q}$ (which one of the many different intensities?) and the distribution length *a*.

Only in the simple particular case of $\vec{q}(s) = \vec{q} = \overrightarrow{constant}$, i.e. all the intensity vectors are parallel, with the same directional sense and the same magnitude *q*, the process of distribution described above leads to the equation *Q* = *qa*. Moreover, as the distribution is along a curve, the center *C* should be additionally found as a point of application of the concentrated force $\vec{Q}$ that is also parallel to $\vec{q}$ and has the same directional sense.

And, the simplest and simultaneously the most popular case is the one of uniform distribution with $\vec{q}(s) = \vec{q} = \overrightarrow{constant}$ along a straight-line segment with a length *a*, and $\vec{q}$ is situated e.g. transversely to the segment (Fig. 3b). Then, not only the equation *Q* = *qa* is valid but also the directrix of $\vec{Q}$ is known: it passes through the midpoint of *a*.

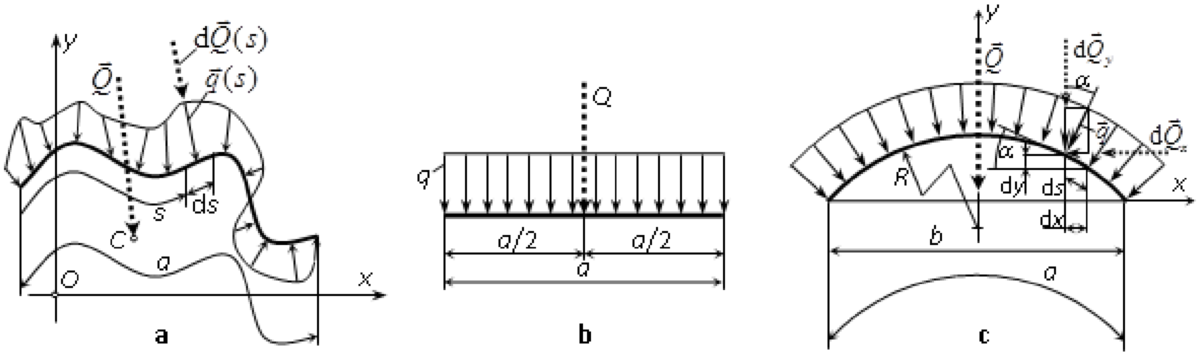

Figure 3: Illustrations to the concentration of distributed forces

Thus, the following becomes clear: in case the distribution is uniform with *q* = *constant* but in radial directions along a circular arc (Fig. 3c, associated with belt drives etc.), the equation *Q* = *qa is not valid* since the vector $\vec{q}$ has a variable direction. Then, it is relevant to resolve each intensity vector $\vec{q}$ into components along the chord of the arc (taken as *x* axis in Fig. 3c) and its perpendicular bisector (*y* axis). It is close to mind that the forces $\mathrm{d}\vec{Q}_x$ from the two sides of the perpendicular bisector balance each other, and $\vec{Q}$ lies on the bisector after summation (integration) of the forces $\mathrm{d}\vec{Q}_y$ only. And, since $\mathrm{d}Q_y = q\mathrm{d}s.\cos\alpha = q\mathrm{d}x$, the integration gives *Q* =

qb: the intensity is not multiplied by the length *a* of the arc but by the length *b* of its chord.

Hence, in case the arc is a semi-circumference, then the intensity will be multiplied by the diameter: namely the equations $N_B = q_{N_B}.2R$ and $\Phi = q_\Phi.2R$ apply to Fig. 1 and Fig. 2.

Usually, in the mind of the students taken the course of Theoretical Mechanics, the equation $Q = qa$ sticks for the simplest and popular case presented in Fig. 3b. Therefore, the engineers are often inclined to multiply the intensity *q* by the distribution length also in all the other cases of uniform distribution i.e. of constant *q* magnitude, without taking into account that the vector $\vec{q}$ has a variable direction. Thus, the above opposing statement even appeared that the distributed force N_B acts actually onto the half of the wheel circumference, i.e. that we are mistaken to think the distribution is along the diameter, and therefore we multiply the intensity q_{N_B} times the diameter $2R$ instead of the half of the wheel circumference.

By the way, in the case of Fig. 3c it stands close to mind that the equation $Q = qa$ cannot be true since it will become $Q = q2\pi R$ when the circular arc becomes a complete circumference. Whereas, then the equation $Q = 0$ is true (the forces $\mathrm{d}\vec{Q}$ balance each other in all the radial directions).

By the way again, subjects like Fig. 3c and the equation $Q = qb$ are taught to students and can be found in (Stefanov 2014) and other textbooks. Respectively, we did not consider our equations $N_B = q_{N_B}.2R$ and $\Phi = q_\Phi.2R$ to necessitate any grounding and proof after their basis is subject to teaching. However, now, in favor of the whole INNO audience, the present paper gives the opportunity to clarify the equation $Q = qb$ and answer why it is valid instead of $Q = qa$.

1.1. Every concentrated force is imaginary

It is emphasized here that every concentrated force is imaginary. It is formed to replace imaginarily the true (original) distributed force in a statically equivalent way and to participate in equilibrium equations: only concentrated forces can participate in them. With that, following the above rules of obtaining the concentrated resultant force, it can find its position in an „empty space”: out of the actual area of application of the original distributed force. An elementary example for this is the following one: the imaginary concentrated force of gravity of a homogeneous ring in a horizontal position passes in the „empty space” through the geometrical center of the ring. Whereas, the original state of the gravity force is: a force distributed onto all the material particles of the very ring.

Thus, nothing is unclear in Fig. 1 and Fig. 2 concerning the position of the traction force *T* at a distance *f* from the driving wheel center i.e. out of the places of contact between the cutting tool and the wheel periphery. That concentrated force *T* is imaginary and it normally finds itself in the „empty space”. The true traction force is the distributed friction force with intensity $\tau(\varphi)$ and it finds itself namely along the semi-circumference of the contact between the cutting tool and the wheel periphery.

2. SUBCRITICAL OR ZERO WORKING COEFFICIENT OF FRICTION

One of the basic points in Theoretical Mechanics (Stefanov 2014) is, as follows. When a static (kinetostatic in our case) equilibrium is enabled with the help of a friction force *T*, then the critical (of the possible maximum, boundary, borderline) *T* magnitude is $\mu_0 N$ were μ_0 is called coefficient of friction at rest (static coefficient of friction), and *N* is the normal (compressive) force which is necessary for appearance of T. The „rest" is understood as „relative rest" i.e. no relative sliding occurs between two bodies which compress each other with N and cohere (mesh) by the friction. With that, the friction (cohesion, meshing) *is expected to be subcritical* (subboundary): $T < \mu_0 N$. Then, the very T force *is determined from the equations of equilibrium*, and the coefficient μ_0 is taken into account insofar as to serve for checking whether the determined force T really satisfies the inequality $T < \mu_0 N$. And, if it proves that the equilibrium requires $T > \mu_0 N$, then this equilibrium will be impossible: relative sliding will occur. If it proves that $T = \mu_0 N$, then the friction, respectively the cohesion, is critical: on the borderline of its ability to hold the equilibrium.

Thus, in the case of subcritical friction, the inequality $T/N < \mu_0$ is valid, and the ratio T/N can be called *working* coefficient of friction (of cohesion), while μ_0 is *critical* (boundary) coefficient of friction (of cohesion). In (Stefanov 2013), the working coefficient of friction was labeled with the simplest possible symbol μ ($\mu = T/N$). Simultaneously, a warning was given that, in Theoretical Mechanics, the same symbol means coefficient of friction at sliding and it should not interfere with the working coefficient of friction introduced. Later, in the textbook (Stefanov 2014) and now also in this paper, a subscript $_\rho$ was preferred to be added to the working coefficient of friction: $\mu_\rho = T/N$ (why the symbol ρ was selected for the subscript can be seen in the textbook mentioned). Correspondingly, $T = \mu_\rho N$ is valid.

Thus, $\mu_\rho < \mu_0$ applies to subcritical friction. With that, μ_ρ can drop to zero in case the equilibrium does not require at all the appearance of a friction force $T = \mu_\rho N$. For example, the blade in Fig. 1 and Fig. 2 moves together with the driven wheel without any need to mesh with it by means of friction: $\mu_\rho = 0$ is valid there (we neglect the very little necessary cohesion for surmounting dissipative resistance of the wheel bearing, air turbulence, etc.). Whereas, the driving wheel should mesh with the blade and produce the traction force that surmounts the cutting force P, and thus the non-zero working coefficient of friction appears according to Eq. 2. The same coefficient becomes zero when $P = 0$ (logarithm of 1 appears then, which gives zero): no frictional meshing of the blade with the driving wheel is necessary, either.

During the preliminary discussions on (Atanasov 2014), misunderstandings appeared about the term „coefficient of friction". In case friction is discussed, it often turns out that the engineers are inclined to imagine only the non-zero friction coefficient μ_0 given in handbooks. And it is not always understood well that μ_0 remains aside as a *critical*, controlling coefficient, whereas the "acting", working friction coefficient μ_ρ is subcritical and is subject to determination, and can be zero in particular. The zero value of μ_ρ does not mean the ability of friction (meshing) disappears but means it is not necessary.

In this chain of thought, we consider as misunderstanding the above objection "if assuming the working coefficient of coherence to be $\mu = 0$, then driving the blade will not be enabled". The working coefficient of coherence is not assumed but is calculated. Driving the blade will not be enabled only if any ability of friction is missing, i.e. if the critical friction coefficient μ_0 is reduced to zero due to ideal smoothness (and/or lack of any adhesion).

3. APPLICATION OF L'HÔPITAL'S RULE FOR OBTAINING X_0

As already understood, we label X as X_0 in case $P = 0$ i.e. when the blade is in the state of idle running (without cutting). Then, already understood again, μ_ρ = tgα = 0: the friction force T in Fig. 1 and Fig. 2 is absent, the force N_A takes its stand onto the straight line AB and takes the magnitude $N_A \equiv N_{A,0} = N_B \equiv N_{B,0} = 2X_0 - \Phi$. The same internal tensile force $N_x = constant = X_0$ is set along the entire blade length. This X_0 can be calculated by Eq. 1 after setting $P = 0$ there. However, it is to notice that the addend PR/μ_ρ in the square brackets transforms into the indeterminate form [0/0]. For its resolution, the well-known L'Hôpital's rule is applied: the numerator and denominator of PR/μ_ρ are substituted with their derivatives with respect to the symbol P.

The derivative of PR with respect to P is R. The derivative of μ_ρ is obtained by differentiating Eq. 2 with respect to P:

$$\frac{\mathrm{d}\mu_\rho}{\mathrm{d}P} = \frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{1}{\pi}\ln\left(1+\frac{P}{X-\Phi/2}\right)\right]$$

$$= \frac{1}{\pi}\frac{1}{1+\frac{P}{X-\Phi/2}}\frac{1}{X-\Phi/2}$$

. Here, P and X are substituted by 0 and X_0: $\frac{\mathrm{d}\mu_\rho}{\mathrm{d}P} = \frac{1}{\pi(X_0-\Phi/2)}$. Thus, it turns out that $PR/\mu_\rho \to [0/0] \to \pi R(X_0 - \Phi/2)$. Then, Eq. 1 gives

$$X \equiv X_0 = \frac{1}{2d+\pi R+\frac{4Ebs}{c}}\left[-PR(X_0-\Phi/2)-\frac{\pi R\Phi}{2}+2Ebs\lambda_s-Ebs\Delta l_\theta-Ebsl\alpha_t\Delta t+\frac{2Ebs\Phi}{c}\right]$$

The expression $\pi R\Phi/2$ participates at two places and is subject to cancelling out. A couple of additional mathematical manipulations should also be done. After all, we have

$$X_0 = \frac{1}{l+\frac{4Ebs}{c}}\left[2Ebs\lambda_s-Ebs\Delta l_\theta-Ebsl\alpha_t\Delta t+\frac{2Ebs\Phi}{c}\right]. \quad \textbf{(3)}$$

Hence, after setting $\Delta l_\theta = 0$, $\Phi = 0$ and $\Delta t = 0$, $X_0 \equiv X_M$ can be obtained: the mounting (assembly) tension of the blade before tilting a wheel, setting in motion and heating. The comparison of X_M, obtained in this way, to the tension which the respective bandsaw indicator shows, is an interesting issue (what actually has the bandsaw designer set to be measured and shown by the indicator?). We intend to study this issue and find the answer.

CONCLUSIONS

1. A unified equation for X has been proposed for both a vertical and a horizontal bandsaw. In that equation, an additional added value participates for taking into account a tilt of the driven wheel (and of the driving wheel if it is also tilted). A better symbol has been introduced for the working coefficient of friction between the blade and the driving wheel.
2. The following points of the solved problem for X have been clarified in details:
   - 2.1. The concentration of a force distributed uniformly and in radial directions along a semi-circumference.
   - 2.2. The subcritical (sub-boundary) friction and the sense of the working coefficient of friction introduced and determined as a part of the problem for X.

2.3. The application of L'Hôpital's rule for determination of $X \equiv X_0$ when $P = 0$ (uniform motion of the blade without cutting).

3. The statically indeterminate problem for X, as set and solved, is useful for clarification of the blade mechanics and gives a new perusal of that mechanics in comparison to the other existing books and papers.

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