MATHEMATICAL DESCRIPTION OF THE CHANGE IN THE ATMOSPHERIC TEMPERATURE DURING DAYS AND NIGHTS

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ABSTRACT

A mathematical description of the periodically changing during many days and nights temperature of the atmospheric air as a processing medium has been suggested. It is introduced in the boundary conditions of our own mutually connected 2D non-linear mathematical models of the logs' freezing and defrosting processes. For the numerical solving of the models a software program has been prepared in the calculation environment of Visual Fortran Professional. Results from a simulative investigation of the 2D non-stationary temperature distribution in beech logs with a diameter of 0.24 m, length of 0.48 m, moisture content of 0.6 kg·kg^{-1}, and initial temperature of 0 ºC during their 5 days and nights continuous freezing and defrosting at sinusoidal change of the air temperature with initial values of –5 ºC and different amplitudes are presented and analyzed.

Key words: mathematical description, atmospheric air, temperature, 2D models, beech logs, freezing, defrosting.

INTRODUCTION

It is known that the duration and the energy consumption of the thermal treatment of frozen logs in the winter, aimed at their plasticizing for the production of veneer, depend on the degree of the logs' icing (Shubin 1990, Požgai et al. 1997, Trebula and Klement 2002, Videlov 2003, Pervan 2009, Deliiski 2009, 2013, Deliiski and Dzurenda 2010). In the accessible specialized literature there are reports about the temperature distribution in subjected to defrosting frozen logs only at conductive boundary conditions (Steinhagen 1986, 1991, Khattabi and Steinhagen 1993, 1995, Deliiski 2009, 2011, Hadjiski and Deliiski 2016).

The modelling and the multi-parameter study of the mutually connected freezing and defrosting processes of logs at atmospheric temperatures are of considerable scientific and practical interest. For example, as a result of such a study it is possible to determine the real icing degree of logs depending on their dimensions, wood species, moisture content, and on the temperature of the air near the logs during their many days staying in an open warehouse before the thermal treatment in the production of veneer. The information about the real value of the icing degree can be used for scientifically based computing of the optimal, energy saving regimes for thermal treatment of each specific batch of logs.

The aim of the present work is to solve two own mutually connected 2-dimensional mathematical models of the transient non-linear heat conduction in logs during their freezing and defrosting at convective boundary conditions with periodically changing atmospheric temperature in the winter and to study the change in the resulting 2D non-stationary temperature distribution and icing degree of beech logs above the hygroscopic range.

BOUNDARY CONDITIONS OF OWN 2D MATHEMATICAL MODELS OF LOGS' FREEZING AND DEFROSTING PROCESSES

In (Deliiski and Tumbarkova 2019) two mutually connected 2D non-linear mathematical models of the logs' freezing and subsequent defrosting processes are given. These models have the following convective boundary conditions, in which the temperature of the processing air medium, T_m, as a variable participates:

• along the radial coordinate r on the logs' frontal surface during the freezing process:

$$\frac{\partial T(r,0,\tau)}{\partial r} = -\frac{\alpha_{\text{wp-fr}}(r,0,\tau)}{\lambda_{\text{wp}}(r,0,\tau)}\left[T(r,0,\tau) - T_{\text{m}}(\tau)\right] \text{ at } T_{\text{m}} < 273.15\ \text{K}, \quad (1)$$

• along the longitudinal coordinate z on the logs' cylindrical surface during the freezing process:

$$\frac{\partial T(0,z,\tau)}{\partial z} = -\frac{\alpha_{\text{wr-fr}}(0,z,\tau)}{\lambda_{\text{wr}}(0,z,\tau)}\left[T(0,z,\tau) - T_{\text{m}}(\tau)\right] \text{ at } T_{\text{m}} < 273.15\ \text{K}, \quad (2)$$

• along the radial coordinate r on the logs' frontal surface during the thawing process:

$$\frac{\partial T(r,0,\tau)}{\partial r} = -\frac{\alpha_{\text{wp-dfr}}(r,0,\tau)}{\lambda_{\text{wp}}(r,0,\tau)}\left[T(r,0,\tau) - T_{\text{m}}(\tau)\right] \text{ at } T_{\text{m}} \geq 273.15\ \text{K}, \quad (3)$$

• along the longitudinal coordinate z on the logs' cylindrical surface during the thawing process:

$$\frac{\partial T(0,z,\tau)}{\partial z} = -\frac{\alpha_{\text{wr-dfr}}(0,z,\tau)}{\lambda_{\text{wr}}(0,z,\tau)}\left[T(0,z,\tau) - T_{\text{m}}(\tau)\right] \text{ at } T_{\text{m}} \geq 273.15\ \text{K}, \quad (4)$$

where T is the temperature, K; τ – time, s; λ_{wp}, λ_{wr} – thermal conductivity of the wood on the logs' surfaces in longitudinal and radial directions respectively, $W \cdot m^{-1} \cdot K^{-1}$; $\alpha_{wp\text{-}fr}$, $\alpha_{wr\text{-}fr}$, $\alpha_{wp\text{-}dfr}$, and $\alpha_{wr\text{-}dfr}$ – convective heat transfer coefficients between the logs and surrounding air during the freezing and defrosting processes, $W \cdot m^{-2} \cdot K^{-1}$.

In (Deliiski and Tumbarkova 2019) mathematical descriptions of all variables in the models are given but the description of T_m only for the case of the freezing of logs in a freezer and of their subsequent defrosting at a room temperature is there presented.

MATHEMATICAL DESCRIPTION OF THE CHANGE IN THE ATMOSPHERIC TEMPERATURE

For numerical solving of the mentioned above mutually connected mathematical models of the logs' freezing and defrosting processes it is needed to have a mathematical description of the temperature of the air medium of the surrounding environment in winter near the logs, T_m.

The periodic change of the atmospheric temperature T_m during the time at a constant value of its amplitude T_{ma} can be described by the following equation (Gusenda and Ganowicz 1986, Deliiski 1988):

$$T_{m} = T_{m0} + (T_{ma} - T_{m0}) \cdot \sin(\omega \cdot \tau), \quad \textbf{(5)}$$

where T_{m0} is the initial value of T_m, K; T_{ma} – amplitude's value of T_m, K; ω – angular frequency of change in T_m, s^{-1}; τ – time, s. The angular frequency of T_m in eq. (5) is equal to

$$\omega = \frac{2\pi}{\tau_0}, \quad \textbf{(6)}$$

where τ_0 is the period of the change in T_m, s. For the precise solving of tasks with the participation of eqs. (5) and (6) it is needed to use $\pi = 3.14159$.

For a periodic change of the air temperature during one day and night, i.e. at $\tau_0 = 1$ d = 24 h = 86400 s, according to eq. (6) it is obtained that

$$\omega = \frac{2\pi}{\tau_0} = \frac{2 \cdot 3.14159}{86400} = 7.2722 \cdot 10^{-5}\ \mathrm{s}^{-1}.$$

When the amplitude of T_m gradually increases or decreases during the time compared to its initial value, $T_{ma\text{-}in}$, then the temperature T_m can be calculated according to the equation

$$T_{m} = T_{m0} + [(T_{ma\text{-}in} - T_{m0}) \cdot (1 \pm K_{ma} \cdot \tau)] \cdot \sin(\omega \cdot \tau), \quad \textbf{(7)}$$

where K_{ma} is a coefficient equal to

$$K_{ma} = \frac{\dfrac{\Delta T_{ma\text{-}\tau_0}}{T_{ma\text{-}in} - T_{m0}}}{\tau_0}, \quad \textbf{(8)}$$

and $\Delta T_{ma-\tau_0}$ is the change in T_{ma} during one period of τ_0, K; $T_{ma\text{-}in}$ – initial value of the amplitude, K.

The signs " + " and " – " in the right side of eq. (7) are used when the amplitude T_{ma} increases or decreases respectively during the periodically change in T_m.

If for example the initial value of $T_{ma\text{-}in}$ is equal to 293.15 K, T_{m0} = 273.15 K, and that value changes by $\Delta T_{ma-\tau_0}$ = 2 K during each period of τ_0 = 1 d = 86400 s, then according to eq. (8) it follows that

$$K_{ma} = \frac{\dfrac{2}{20}}{86400} = 1.15741 \cdot 10^{-6}.$$

RESULTS AND DISCUSSION

The mathematical description of the periodically changing atmospheric temperature given above was introduced in the boundary conditions (1) to (4) of the mutually connected mathematical models of the logs' freezing and defrosting processes.

For the numerical solving of the models, a software program was prepared in the calculation environment of Visual FORTRAN Professional developed by Microsoft.

With the help of the program computations were made for the determination of the 2D non-stationary change of the temperature in the longitudinal sections of two beech logs named below as Log 1 and Log 2. The logs were with a diameter D = 240 mm, length L = 480 mm, basic density ρ_b = 560 kg·m^{-3}, moisture content u = 0.6 kg·kg^{-1}, and initial temperature t_{w0} = 0 °C. Two options of 120 h continuous periodic freezing and defrosting of the logs have been studied as follows:

- for Log 1: at constant values of t_{m0} = –5 °C and t_{ma} = 20 °C;
- for Log 2: at constant value of t_{m0} = –5 °C and at gradual decreasing of the amplitude's value $t_{ma\text{-}in}$ = 20 °C by 2 °C/d.

During the solving of the models, the mathematical description of the thermophysical characteristics of beech wood with fiber saturation point at 293.15 K (i.e. at 20 °C) $u_{\mathrm{fsp}}^{293.15} = 0.31\ \mathrm{kg \cdot kg^{-1}}$ was used. At that value of $u_{\mathrm{fsp}}^{293.15}$ the studied logs contain a maximum possible bound water, equal to $u_{\mathrm{fsp}}^{272.15}$ = 0.331 kg·kg^{-1}. (Deliiski 2013). This means that at u = 0.6 kg·kg^{-1} the logs contain a free water, equal to 0.6 – 0.331 = 0.269 kg·kg^{-1}.

The models have been solved with the help of explicit schemes of the finite difference method in a way, analogous to the one used and described in (Deliiski 2009, 2011, Deliiski and Tumbarkova 2019) for the solution of a model of the heating process of cylindrical wood materials. For this purpose, the calculation mesh has been built on ¼ of the longitudinal section of the logs due to the circumstance that this ¼ is mirror symmetrical towards the remaining ¾ of the same section.

The models were solved with step $\Delta r = \Delta z$ = 0.006 m along the coordinates r and z. This means that the number of the steps along r was 20 and along z it was 40, i.e. the total number of the knots in the logs' longitudinal section was equal to N_{total} = 20 x 40 = 800. The interval between the time levels, Δτ, (i.e. the step along the time coordinate), has been determined by the software according to the condition of stability for explicit schemes of the finite difference method (Deliiski 2011) and in our case it was equal to 6 s.

On Figure 1 the coordinates of 4 characteristic points in the logs are given, in which the calculated change in the temperature t was registered and graphically presented. Point 1 is with r = 30 mm and z = 120 mm; Point 2: with r = 60 mm and z = 120 mm; Point 3: with r = 90 mm and z = 180 mm and Point 4: with r = 120 mm and z = 240 mm (center of the log).

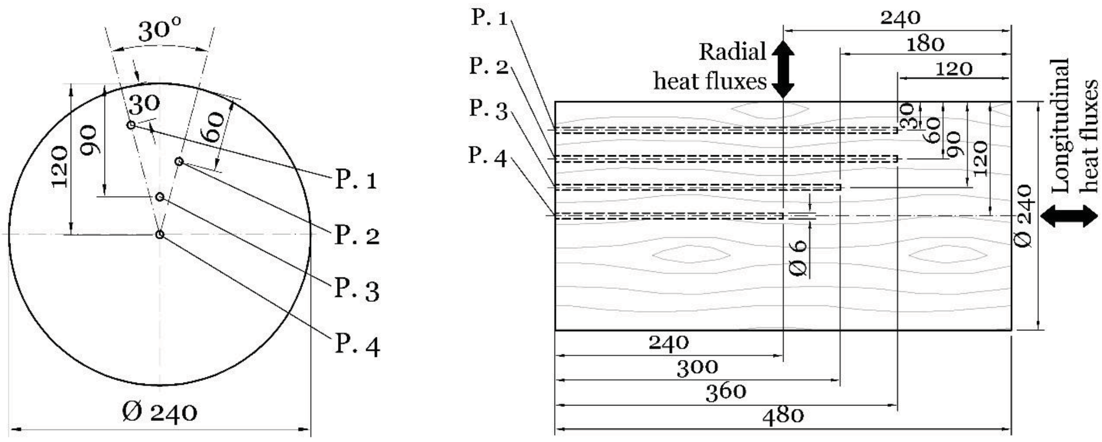

Figure 1: Radial (left) and longitudinal (right) coordinates of 4 characteristic points for registration of the temperature change in logs subjected to periodically freezing and defrosting

These coordinates of the characteristic points allow for the determination and analyzing of the 2D temperature distribution in logs during their freezing and defrosting.

Figure 2 and Figure 3 present the calculated change in temperature of the processing air medium, t_{m}, logs' surface temperature, t_{s}, logs' average mass temperature, t_{avg}, and t of 4 characteristic points in the studied beech logs during their periodic freezing and defrosting at constant values of t_{m0} = –5 °C and t_{ma} = 20 °C (Log 1) and at constant value of t_{m0} = –5 °C and at gradual decreasing of the value of the amplitude $t_{\mathrm{ma\text{-}in}}$ = 20 °C by 2 °C/d (Log 2) respectively.

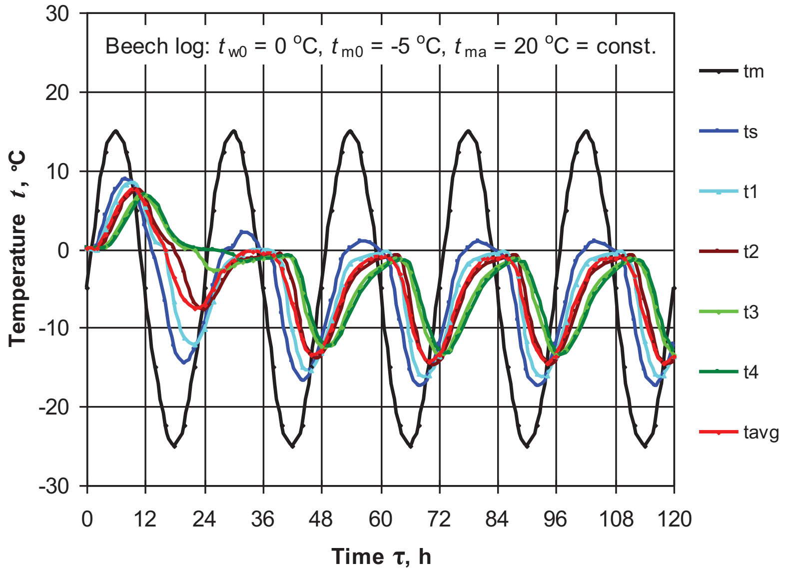

Figure 2: Calculated with the model change in t_m, t_s, t_{avg}, and t of 4 characteristic points of the Log 1 during its 120 h periodically freezing and defrosting at constant values of t_{m0} and t_{ma}

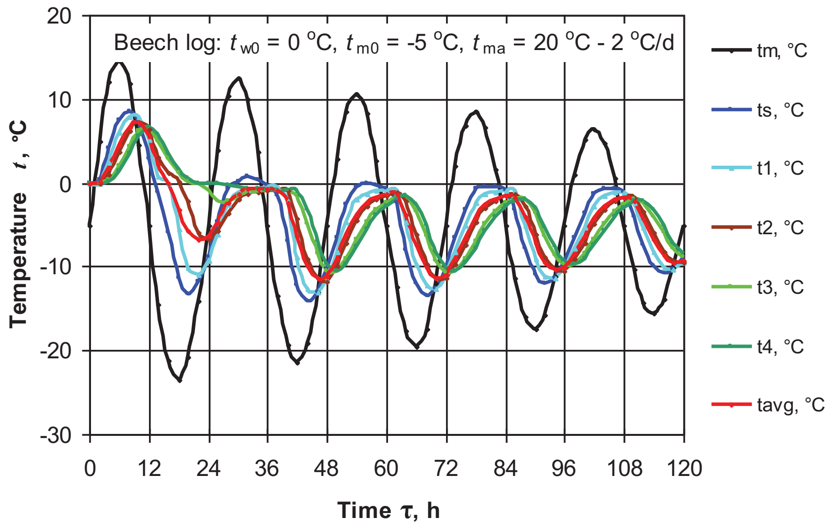

Figure 3: Calculated with the model change in t_m, t_s, t_{avg}, and t of 4 characteristic points of the Log 2 during its 120 h freezing and defrosting at constant value of t_{m0} and decreasing values of t_{ma} by 2 °C/d

On Figure 2 it can be seen that at constant values of t_{m0} = –5 °C and t_{ma} = 20 °C after 72nd h, i.e. after the 3rd period of t_m, a periodical change in the log's temperature with practically constant amplitudes for the separate points is coming. As far as the point is distanced from the logs' surfaces that much smaller is the amplitude of the periodic change of the temperature in that point. The amplitudes of t_m and t in the separate characteristic points after the 3rd period are equal to as follows: t_{ma} = 20.0 °C, t_{sa} = 9.5 °C, t_{1a} = 8.2 °C, t_{2a} = 7.2 °C, t_{3a} = 6.5 °C, and t_{4a} = 6.0 °C.

When t_{m0} remains constant and t_{ma} decreases during the time, the amplitudes of t in the separate characteristic points also gradually decrease (see Figure 3). At t_{m0} = –5 °C = const and t_{ma} = 20 °C – 2 °C/d, during the 5th period t_{ma} = 12.0 °C, t_{sa} = 5.3 °C, t_{1a} = 4.8 °C, t_{2a} = 4.5 °C, t_{3a} = 4.2 °C, and t_{4a} = 4.0 °C.

The average mass temperature of the logs, t_{avg}, at 120th h is equal to –13.56 ^{o}C for Log 1 and to –9.23 ^{o}C for Log 2.

Synchronously with the determination of the 2D non-stationary change of the temperature in the longitudinal sections of the studied logs, calculations of the change in their total icing degree, $\Psi_{ice\text{-}total}$, during the periodically freezing and defrosting have been carried out. These calculations were made according to the complex algorithm containing 9 steps, which are detailed in (Deliiski and Tumbarkova 2017).

Figure 4 presents the change in the icing degree $\Psi_{ice\text{-}total}$ of the studied logs, which is caused by the freezing of both the free and bound water in them.

During the first 2 h of the 1st period of t_m the icing degree $\Psi_{ice\text{-}total}$ increases from 0 to 0.07 due to the freezing of the free water only in some peripheral layers of the logs. From 2nd to 12th h of the 1st period $\Psi_{ice\text{-}total}$ is equal to 0 because then the whole amount of the free and bound water in the logs is in a liquid state.

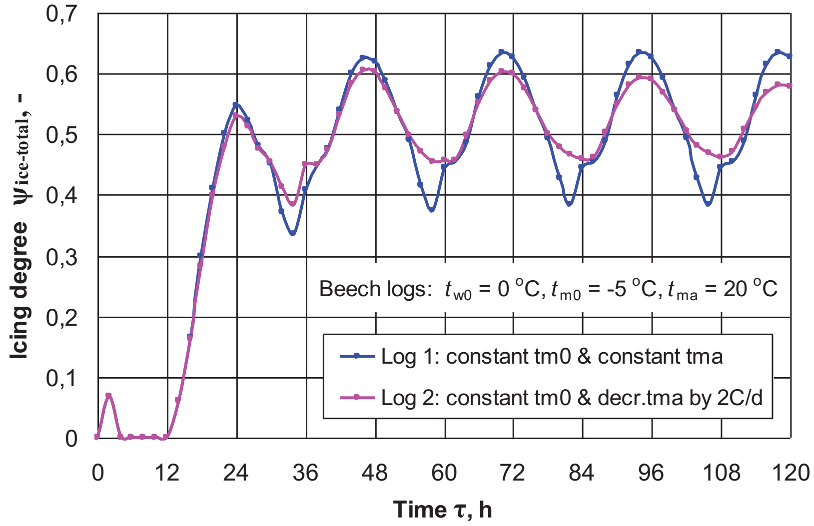

Figure 4: Change in $\Psi_{ice\text{-}total}$ during the periodically freezing and defrosting of the studied logs

During the second half of the 1st period $\Psi_{ice\text{-}total}$ increases from 0 to 0.55 for Log 1 and from 0 to 0.53 for Log 2. These values of $\Psi_{ice\text{-}total}$ mean that 55% (i.e. 0.33 kg·kg^{-1}) from the whole amount of the moisture content of 0.6 kg·kg^{-1} for Log 1 and 0.53% (i.e. approximately 0.32 kg·kg^{-1}) for Log 2 are then in frozen state.

After the 1st period the change in $\Psi_{ice\text{-}total}$ is periodical and during the last 5th period of t_m it is in the range from 0.38 to 0.64 for Log 1 and for 0.46 to 0.58 for Log 2. At the end of the 5th period $\Psi_{ice\text{-}total}$ = 0.627 for Log 1 and $\Psi_{ice\text{-}total}$ = 0.579 for Log 2. These values of $\Psi_{ice\text{-}total}$ mean that 62.7% (i.e. 0.376 kg·kg^{-1}) for Log 1 and 57.9% (i.e. 0.347 kg·kg^{-1}) for Log 2 from the whole amount of the moisture content of 0.6 kg·kg^{-1} are then in a frozen state.

The rest amounts of the moisture content u = 0.6 kg·kg^{-1}, i.e. 37.3% (i.e. 0.224 kg·kg^{-1}) for Log 1 and 42.1% (i.e. 0.253 kg·kg^{-1}) for Log 2 are in a liquid state in the cell walls of the logs at the end of 120 h periodic freezing and defrosting of the studied beech logs.

CONCLUSIONS

This paper presents a methodology for mathematical modeling and research of two mutually connected problems: 2D non-stationary temperature distribution in logs subjected to many days and nights freezing and defrosting at periodically changing air temperature near them in winter and change in the icing degree of the logs during these processes.

A mathematical description of the periodically changing atmospheric temperature in winter has been carried out. That description is introduced in our own mutually connected 2D non-linear mathematical models of the 2D temperature distribution in logs during their freezing and defrosting.

With the help of the model, computations for the determination of the temperature distribution and icing degree have been completed in calculation environment of Visual FORTRAN Professional for of two beech logs with a diameter of 0.24 m, length of 0.48 m, moisture content of 0.6 $kg \cdot kg^{-1}$, and initial temperature of 0 ºC, during their 5 days and nights continuous freezing and defrosting.

Two options of the periodic sinusoidal change in the air temperature near the logs have been studied: at constant initial value of –5 ºC and constant amplitude of 20 ºC for Log 1 and at constant initial value of –5 ºC and decreasing amplitude of 20 ºC by 2 ºC/d for Log 2.

It has been computed that after 120 h of the studied processes the total icing degree, $\Psi_{ice\text{-}total}$, of the logs reaches the following values: $\Psi_{ice\text{-}total}$ = 0.627 for Log 1 and $\Psi_{ice\text{-}total}$ = 0.579 for Log 2. This means that from the total amount of free and bound water in the logs, equal to 0.6 $kg \cdot kg^{-1}$, in a frozen state then are 0.376 $kg \cdot kg^{-1}$ in Log 1 and 0.347 $kg \cdot kg^{-1}$ in Log 2.

The solution of the models allows for the calculation of the temperature distribution and icing degrees, and also different energy characteristics of logs from diverse wood species for each desired moment during their freezing and defrosting at periodically changing atmospheric temperature with specific parameters. The results from the solutions can be used for the development of scientifically based energy saving optimized regimes for thermal treatment of frozen logs with consideration of their specific icing degree and also in the software of systems for model predictive automatic control (Hadjiski and Deliiski 2016) of that treatment.

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