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MODELING OF THE FREE SPATIAL VIBRATIONS OF WOOD SHAPER AND ITS SPINDLE

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ABSTRACT

The proposed study deals with the modeling of the free spatial vibrations of a woodworking shaper and its spindle. An original mechanical - mathematical model of woodworking shaper and its spindle, developed by the authors, is presented in this work. The model provides the opportunity to explore the free undamped space vibrations of this type of machinery. In this model the woodworking shaper and its spindle are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor. A system of matrix differential equations is compiled and analytical solutions are presented. The natural frequencies and mode shapes for a specific machine can be obtained with their help. The model is supplemented in order to investigate the free damped space vibrations of the considerate system. The new model takes into account the damping properties of machine's elements. A new system of matrix differential equations is developed and the relevant analytical solutions are presented. They allow various numerical solutions for specific machines. The models take into account the characteristics in the construction of woodworking shapers. They include all necessary geometric parameters of this system. Numerical solutions and graphs can be obtained with the help of the developed models. These solutions and graphs are necessary for analyzing the free undamped and the free damped space vibrations of a particular considerate machine.

Key words: wood shapers, free vibrations, modeling.

INTRODUCTION

The examination of the trachea as ab Reduction of the level of vibrations and noise of the woodworking shapers implies the introduction of certain requirements for the construction and the manner of it works (Gochev and Vukov 2017, Vukov et al. 2016). In terms of the theory of mechanical vibrations, it means to formulate specific measures and manners to influence the vibrating system to which this machine is commuted (Coutinho 2001, Wittenburg 1977). The main task of each study is to replace the real machine with a corresponding vibrating system. Solving of this task requires adequate mechanic-mathematical modelling and compiling the system equations describing the vibrations of the woodworking shapers elements (Amirouche 2006, Angelov and Slavov 2010). Well-targeted studies can be carried out by solving these equations under different assumptions. Some grounded recommendations for the construction and the manner of operation of the woodworking shapers are formed, based on these studies (Beljo et al. 2001, Kminiak et al. 2016, Orlowski et al. 2007).

The vibration characteristics of the woodworking shapers depend on their natural vibration frequencies as well as any mechanical vibration system. It is known that when the frequencies of the external influences, which cause vibrations, are equal to a frequency of their natural frequencies appears the phenomenon „resonance“. Resonance regimes can lead to significant increase of vibration amplitudes. Significant vibration amplitudes change the normal work regimes of the wood shapers and damage the accuracy and quality of the production (Vukov et al. 2013). Extra stress, which is caused by increase of vibration amplitudes, sometimes can reach such values that might damage or even destroy some machine's elements. This causes reduction of the reliability of the woodworking shapers.

A main rule in the engineer's practice is that resonant regimes are unwilling. They must be avoided by a suitable selection of elements' parameters of the woodworking shapers as well as the machine's work regimes. Therefore, it is necessary to make an assessment of the resonant danger in advance when the woodworking shaper is designed and is dimensioned. Dangerous work regimes during the machine's operation are not allowed. Consequently, the natural frequencies and the mode shapes must be investigated. Therefore, the first task in examining the vibration properties of the machine is to investigate its free undamped vibrations. This study is based on a developed mechanic-mathematical model which allows obtaining the natural frequencies and the mode shapes of the considerate mechanical system.

The work of the woodworking shapers often involves passing through transient regimes. The study of damped vibrations is useful for analyzing the characteristics of these regimes. Therefore, the developed mechanic-mathematical model can be adapted to study also free damped vibrations. Examinations of these vibrations help the selection and the control of technical state of some machine's components. It mainly concerns the vibro-isolators between the machine and the floor and the bearing units of the spindle.

The idea that the woodworking shaper and its spindle are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor, derives from the written above. These elastic elements are four vibration isolators between the machine and the floor and two bearing units of the spindle.

The aim of this work is to build a mechanic – mathematical model of a woodworking shaper and its spindle, which gives the opportunity for exploration the free space vibrations of this type of machinery. The model is designed to study free undamped spatial vibrations of these machines, and then it is adapted to explore their free damped spatial vibrations also. The model refers to wood shapers with lower placement of the spindle. The model renders in account the construction's characteristics of this class of wood shapers.

A kind of woodworking shapers that are commonly used in the practice of forestry industry (Filipov 1977) are examined in the proposed study. Fig. 1 shows the general view, and Fig. 2 – a scheme of this type of woodworking shapers (Obreshkov 1996). The machine body is marked with *1*, *2* is the electric motor, *3* – the belt drive, *4* – the vibration isolators between the machine and the floor, *5* – the spindle with the bearings, *6* – the abror with morse cone, *7* – the work table, *8* – the cutter.

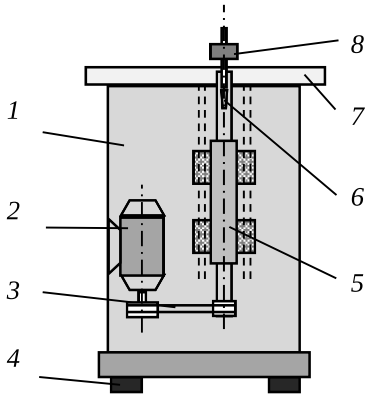

Figure 1: Wood shaper – general view Figure 2: Scheme of wood shaper

Analysis of woodworking shapers construction shows the strong influence of the spindle'' work on the functioning of the whole machine. Fig. 3 shows the general view, and Fig. 4 – the spindle with fitted cutter.

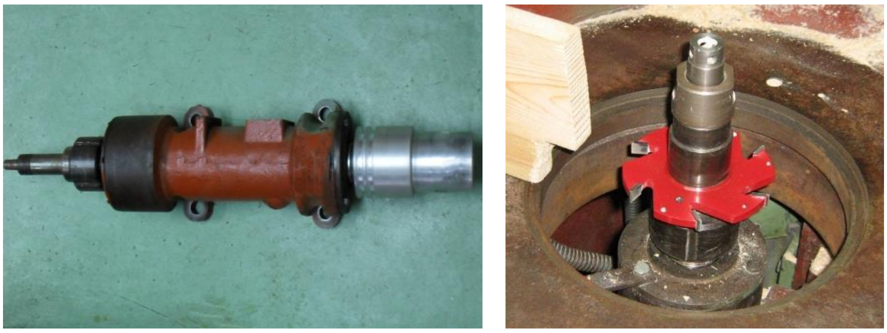

Figure 3: Spindle with bearing units and with cutter

METHODOLOGY AND RESULTS

A mechanical - mathematical model of wood shapers with lower spindle is built for studying its free spatial vibrations. The model is shown in Fig. 4. The wood shaper and its spindle are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor. These elastic elements are four vibration isolators between the machine and the floor, and the two bearing units of the spindle.

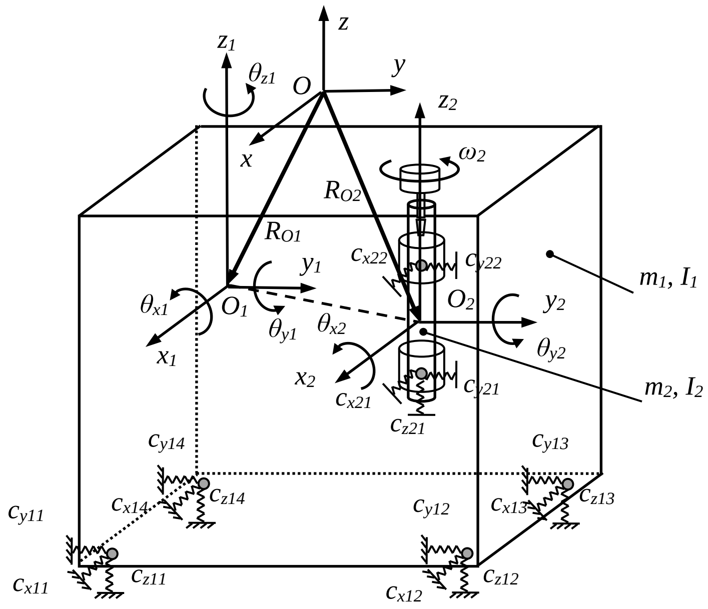

Figure 4: Mechanic-mathematical model for investigation of the free undamped spatial vibrations of the woodworking shaper and its spindle

The following symbols are used:

m_1, m_2 – mass of the wood shaper and its spindle;

$\mathbf{I}_1$, $\mathbf{I}_2$ – inertia moment tensors of the wood shaper and its spindle;

c_{x1i}, c_{y1i}, c_{z1i} , $i = 1, 2, 3, 4$ – elastic coefficients of the vibroisolators between the machine and the floor;

c_{x2i}, c_{y2i}, c_{z2i} , $i = 5, 6$ – elastic coefficients between the machine and the spindle;

This model has 12 degrees of freedom. The vector of the generalized coordinates is

$$\mathbf{q} = \left[x_1 \ y_1 \ z_1 \ \theta_{x1} \ \theta_{y1} \ \theta_{z1} \ x_2 \ y_2 \ z_2 \ \theta_{x_2} \ \theta_{y_2} \ \theta_{z_2}\right]^{\mathrm{T}} . \qquad \mathbf{(1)}$$

The matrixes of the transition in small vibrations between the local coordinate systems of the bodies and the reference coordinate system have the form

$$\mathbf{A_i^0} = \begin{bmatrix} 1 & -\theta_{zi} & \theta_{yi} & x_i \\ \theta_{zi} & 1 & -\theta_{xi} & y_i \\ -\theta_{yi} & \theta_{xi} & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad i = 1, 2 . \qquad \mathbf{(2)}$$

The vector of the position of the center of mass of the relevant body is determined with

$$\mathbf{R_{Ci}^0} = \mathbf{A_i^0}.\mathbf{r_{Ci}} = \begin{bmatrix} l_{Cx} + x_i + l_{Cz}.\theta_{yi} - l_{Cy}.\theta_{zi} \\ l_{Cy} + y_i - l_{Cz}.\theta_{xi} + l_{Cx}.\theta_{zi} \\ l_{Cz} + z_i + l_{Cy}.\theta_{xi} - l_{Cx}.\theta_{yi} \\ 1 \end{bmatrix} \qquad i = 1, 2 , \qquad \mathbf{(3)}$$

where $\mathbf{r_{Ci}} = \begin{bmatrix} l_{Cx} & l_{Cy} & l_{Cz} \end{bmatrix}^{\mathrm{T}}$ is the vector of the position of the center of mass in the local coordinate system.

The vector of absolute linear velocity of the center of mass of the respective body is calculated as follows:

$$\mathbf{V_{Ci}^{0}} = \frac{d\mathbf{R_{Ci}^{0}}}{dt} = \begin{bmatrix} \dot{x}_i + l_{Cz}.\dot{\theta}_{yi} - l_{Cy}.\dot{\theta}_{zi} \\ \dot{y}_i - l_{Cz}.\dot{\theta}_{xi} + l_{Cx}.\dot{\theta}_{zi} \\ \dot{z}_i + l_{Cy}.\dot{\theta}_{xi} - l_{Cx}.\dot{\theta}_{yi} \\ 0 \end{bmatrix} \qquad i = 1, 2 \,. \tag{4}$$

The vector of absolute angular velocity of the respective body, projected in the local coordinate system, has the form

$$\mathbf{\Omega_{i}^{i}} = \begin{bmatrix} \dot{\theta}_{xi} \\ \dot{\theta}_{yi} \\ \dot{\theta}_{zi} \\ 0 \end{bmatrix} \qquad i = 1, 2 \,. \tag{5}$$

The deduction of the kinetic energy and potential energy of the system is convenient to be made with a symbolic method and modern software (Mathematica, MATLAB).
The kinetic energy of the mechanical system is determined with

$$\mathrm{E_K} = \sum_{i=1}^{2} \left(\frac{1}{2} . \begin{bmatrix} \dot{\mathrm{R}}_{\mathrm{Ci}}{}^{\mathrm{T}} & \dot{\Theta}_{\mathrm{i}}{}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} . \begin{bmatrix} \mathrm{m_{RRi}} & \\ & \mathrm{I_{\Theta\Theta i}} \end{bmatrix} . \begin{bmatrix} \dot{\mathrm{R}}_{\mathrm{Ci}} \\ \dot{\Theta}_{\mathrm{i}} \end{bmatrix} \right), \tag{6}$$

where

$$\mathbf{m_{RR\mathit{i}}} = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix};$$

$$\mathbf{I_{\Theta\Theta\mathit{i}}} = \begin{bmatrix} I_{xxi} & -I_{xyi} & -I_{xzi} \\ -I_{xyi} & I_{yyi} & -I_{yzi} \\ -I_{xzi} & -I_{yzi} & I_{zzi} \end{bmatrix},$$

$$\dot{\mathbf{R}}_{\mathbf{C}i} = \begin{bmatrix} \dot{x}_{Ci} & \dot{y}_{Ci} & \dot{z}_{Ci} \end{bmatrix}^{\mathrm{T}};$$

$$\dot{\mathbf{\Theta}}_i = \begin{bmatrix} \dot{\theta}_{xi} & \dot{\theta}_{yi} & \dot{\theta}_{zi} \end{bmatrix}^{\mathrm{T}}.$$

Potential energy is defined by

$$\mathrm{E_P} = \mathrm{E_{P1}} + \mathrm{E_{P2}} = \left(\sum_{k=1}^{4} \frac{1}{2} \mathrm{c_k} . \left(\delta \mathrm{r_k^{01}} \right)^2 + \sum_{k=1}^{2} \frac{1}{2} \mathrm{c_k} . \left(\delta \mathrm{r_k^{12}} \right)^2 \right) + \sum_{i=1}^{2} \mathrm{E_{PGi}} \,, \tag{7}$$

where

$$\boldsymbol{\delta}\mathbf{r_k^{01}} = \mathbf{R_1} + \mathbf{U_1^0}.\mathbf{r_k^{01}} - \mathbf{r_k^{01}},$$

$$\boldsymbol{\delta}\mathbf{r_k^{12}} = \left(\mathbf{R_1} + \mathbf{U_1^0}.\mathbf{r_k^{12}} - \mathbf{r_k^{12}} \right) - \left(\mathbf{R_2} + \mathbf{U_2^0}.\mathbf{r_k^{21}} - \mathbf{r_k^{21}} \right),$$

$\mathbf{R_i} = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}^{\mathrm{T}} \quad i = 1, 2$ – vector of the position of the beginning of the mobile (related with the body) coordinate system relative to the fixed coordinate system,

$\boldsymbol{\delta}\mathbf{r_k^{01}}$ – the deformation of the elastic elements between the base (marked conditionally with "0") and the body 1,

$\boldsymbol{\delta}\mathbf{r_k^{12}}$ – the elastic deformation of the elements between the two bodies.

The differential equations which describe the free vibrations are deduced by using the Lagrange's method. This method provides the best opportunities.

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}}\right)-\frac{\partial E_K}{\partial q}+\frac{\partial E_P}{\partial q}=0\,, \qquad \textbf{(8)}$$

where E_K and E_P are respectively the kinetic and the potential energy of the systems.

The obtained system of parametric differential equations, which describes the free vibrations of the mechanical system, is

$$\mathbf{M}.\ddot{\mathbf{q}}+\mathbf{C}.\mathbf{q}=0 \qquad \textbf{(9)}$$

The natural frequencies and the mode shapes of the mechanical system are obtained after solving the system equations (9).

In order to investigate the free damped spatial vibrations of the woodworking shaper, it is necessary to modify the model, presented above. The woodworking shaper and its spindle are regarded again as rigid bodies, which are connected with each other and with the motionless floor. The damping properties of the connecting elements in addition to their damping properties should be taken into account in the second model. This model accepts the form, shown in Fig. 5.

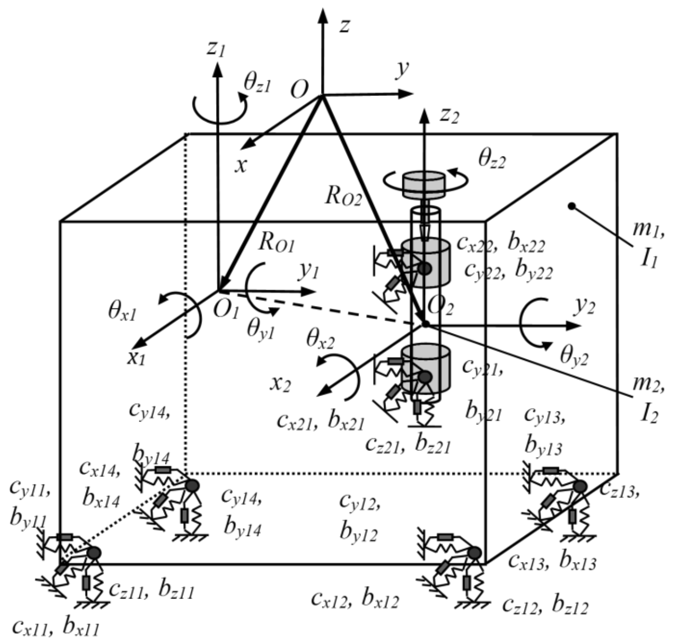

Figure 5: Mechanic-mathematical model for investigation of the free damped spatial vibrations of the woodworking shaper and its spindle

The used symbols are analogous to those of Fig. 5, but the following ones are added:

$\boldsymbol{b_{x1i}}$, $\boldsymbol{b_{y1i}}$, $\boldsymbol{b_{z1i}}$, $i = 1, 2, 3, 4$ – damping coefficients of the vibroisolators between the machine and the floor;

$\boldsymbol{b_{x2i}}$, $\boldsymbol{b_{y2i}}$, $\boldsymbol{b_{z2i}}$, $i = 1, 2, 3, 4$ – damping coefficients between the machine and the spindle.

The differential equations which describe the free damped spatial vibrations are deduced by using the Lagrange's method

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial E_K}{\partial \dot{\mathbf{q}}}\right)-\left(\frac{\partial E_K}{\partial \mathbf{q}}\right)+\frac{\partial F_b}{\partial \dot{\mathbf{q}}}+\frac{\partial E_P}{\partial \mathbf{q}}=0\,. \qquad \textbf{(10)}$$

where F_b is the energy dissipation or dissipative function.

The obtained system of differential equations, which describes the small free damped vibrations of the mechanical system, is

$$\mathbf{M}\,.\ddot{\mathbf{q}}+\mathbf{B}\,.\dot{\mathbf{q}}+\mathbf{C}\,.\mathbf{q}=0\,. \qquad \textbf{(11)}$$

The matrix in these equations which characterizes the mass-inertial properties of the mechanical system is M**,** and the elastic properties – C. $\mathbf{B}(\dot{\mathbf{q}})$ is the matrix that characterizes the damping properties of this system.

$$\mathbf{M}=\left[a_{ij}\right], \qquad a_{ij}=\frac{\partial^2 E_K}{\partial \dot{q}_i\,.\partial \dot{q}_j}, \qquad \textbf{(12)}$$

$$\mathbf{C}=\left[c_{ij}\right], \qquad c_{ij}=\frac{\partial^2 E_P}{\partial q_i\,.\partial q_j}. \qquad \textbf{(13)}$$

The matrix $\mathbf{B}=\left[b_{m,n}\right]$ is obtained by substituting the elements of the matrix **C** – $c_{m,n}$, with $b_{m,n}$.

$$\mathbf{B}=\left[b_{ij}\right], \qquad b_{ij}=\frac{\partial^2 F_b}{\partial \dot{q}_i\,.\partial \dot{q}_j}. \qquad \textbf{(14)}$$

The diagrams of free damped spatial vibrations of the woodworking shaper and its spindle are obtained after solving the system equations (11).

CONCLUSION

The presented study considers the modeling of the free spatial vibrations of a woodworking shaper and its spindle. Two mechanical – mathematical models of woodworking shaper and its spindle, developed by the authors, are presented in this work. The first model provides the opportunity to explore the free undamped space vibrations of this type of machines. In this model the woodworking shaper and its spindle are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor. A system of matrix differential equations is compiled and analytical solutions are presented. The second model provides the opportunity to explore the free damped space vibrations of this system. This model is obtained by taking into account the damping properties of machine's elements additionally. A new system of matrix differential equations is developed and the relevant analytical solutions are presented. The two models allow various numerical solutions for specific machines. Received numerical solutions and graphs are necessary for analyzing the free undamped and the free damped space vibrations of a particular considerate machine.

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